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A common misinterpretation of the equal-area net properties

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INTRODUCTION

THIS contribution draws attention to a misinterpretation of the meaning of the term 'equal-area' when the principal properties of the Schmidt net are described. This error, namely the idea that 2° × 2° squares defined by parallels and meridians have the same surface area all over the net, has been found in several of the most commonly used books on Structural Geology. This error does not lead to wrong manipulation or interpretation of data but is of conceptual importance.

THE MISINTERPRETATION

The Schmidt net is based on a projection presented by Lambert in 1772. This projection is azimuthal and equal-area, so "directions from a central point to any other on the projection are shown correctly" and "areas of all regions are shown on the projection in the same proportion to their true areas" (see Snyder 1987, Snyder & Voxland 1989). The author's teaching experience has shown that students generalize this to mean that "the area of a 2° × 2° square in the border of the net is the same as another 2° × 2° square in its center". This is untrue.

While introducing graphic counting techniques, some textbooks on Structural Geology make the same mistake. A survey over some of the most commonly used textbooks in Structural Geology has rendered the following results.

Books in which the statement is correctly expressed

Turner & Weiss 1963: "All equal areas on the surface of the reference sphere must remain equal on the projection itself". This seems to be one of the simplest and clearest of all definitions.

Whitten 1966: "Equivalent areas on the reference sphere remain equal on the projection".

Ramsay 1967: "The surface area of all 10 great-circle intercepts of any zone between two given small circles on the hemisphere surface are equal. On the stereographic net however they are not of equal area . . . the positions of the grid lines can be adjusted so that the areas are equal. . . . The equal-area Schmidt or Lambert net that results from this correction is the most useful graph for plotting and interpreting structural data".

Hobbs et al. 1981: "Equal areas on the projection sphere are represented by equal areas on the projection plane, although their shapes can be greatly distorted".

Suppe 1985: "Equal areas on the surface of the sphere map onto the image plane as equal areas".

Sellés-Martínez 1988: "Surface areas of 2° × 2° squares on the projection are proportional to their true areas on the spherical surface".

Books in which the statement is ambiguously or misleadingly expressed

Some authors, quoting statements like that of Ramsay (1967), forget the complementary one, and produce ambiguous statements.

Ragan 1968: "An area (say 10° × 10°) in the center of the net is smaller that the same angular area at the margin. To overcome this, an equal area or Schmidt net is used".

Phillips 1977: "An area bounded by four arcs of 10° projected near the centre of the stereogram is much smaller than the same area of surface projected near the primitive" (for the Wulff net). "There is considerable advantage in using in place of the stereographic, a scheme of projection of the sphere which is area-true. The Lambert equal-area projection is customarily used". Putting together both statements leads to the erroneous conclusion that in Lambert equal-area projection the area bounded by four arcs of 10° is the same everywhere.

Ragan 1984: "An area of the net of say, 10° × 10° in the center of the net is smaller than the same angular area at the margin. To overcome this a different type projection is needed. The method used is called the Lambert equal area. . .".

Books in which the statement is wrongly expressed or illustrated

In some textbooks the mistake is evident.

Comite des Techniciens 1976: "La proprieté essentielle du canevas de Schmidt est d'etre construit de telle façon que les surfaces limitées par deux méridiens et deux paralléles soient égales entre elles quelle que soit leur position sur la projection". Figure 60 of this book compares areas in the center of the net and its margin, but limited by the same parallels, so it becomes a correct example of an incorrect statement. Paradoxically this is one of the few books which discusses the change in shape involved in projection close to the border of the net, and changes in apparent density caused by the use of invariable cell counters.

Davis 1984: "The geometry of projection of the Schmidt net is such that 2° areas bounded by great and small circles are the same size across the net".

Marshak & Mitra 1988: "A grid constructed on an equal area projection is called a Schmidt net (. . .). Such a projection does not cause the area of a projected circle to vary with its position although its shape does change". This is undoubtedly true, but ". . . on a Schmidt net the size of a 10° × 10° area near the primitive is the same as that of the center" is still an ambiguous statement. Illustration 8-3 in their book is incorrect. Any of the six 10° × 10° squares they compare lies between the same two parallels.

Spanish translations, when available, have shown to be in accordance with their originals and so they have not been quoted, although they have been surveyed.

DISCUSSION: REASONS FOR MISINTERPRETATIONS

The mistake seems to have two sources, one in the use of the equatorial projection for plotting purposes and the other in the formerly mentioned extrapolation of the properties of the projection to the grid. Due to the plotting technique, only the N–S and E–W scales of the grid are used (so in practice the polar projection is actually being used) and treating the E–W direction on the grid as being representative of the whole reference frame of the equatorial projection is unsound. A 10° × 10° square drawn in the center of the net has the same area as the 10° × 10° square drawn in its western or eastern border, and the same area of a 10° × 10° square drawn on the north pole. However the latter has to be drawn by rotating the tracing paper to the E–W direction. So it has nothing in common with the square defined by two meridians and two parallels in the polar region of the equatorial projection.

The author has found that for teaching purposes the following ways of differentiating the properties of the Wulff and Lambert projections have proved useful.

Circles drawn in the spherical surface at different latitudes project as circles of different radius on the Wulff net but as ellipses of constant area but variable ellipticity in the Schmidt net (Fig. 1).

With respect to the reference frame of the nets it can be demonstrated that the surface of a 2° × 2° square near the equator is not the same shape or area as the surface of a similar square near the poles, as it is not on the Earth's surface (Fig. 2). If we observe on a globe the quadrangle defined by the meridians of Greenwich and 10°W and the Equator and the parallel of 10°N and compare it with the quadrangle defined by the same meridians but between the 80° and 90° parallels, we realize that not only their surface shapes are actually different but also the latter is not a square but a triangle. More importantly the area difference has nothing to do with the Wulff–Schmidt transformation, so when comparing the parallels–meridians reference frame with that of the net, there is no reason for the 2° × 2° cells to have the same area all over the net.

Finally we can make a simple test. If we take a counting net, like the Calsbeek net, and superimpose it on a Schmidt net, we can easily find out that a different number of 2° × 2° cells is enclosed in the 1% area units of the counting net, increasing from the center of the Schmidt net to its poles. Thus the area of the cells of the grid is not constant.

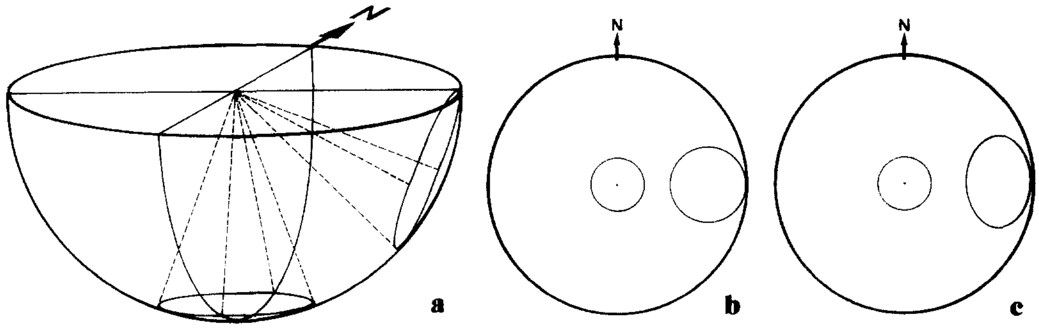

Fig. 1. Properties of stereographic and equal-area projections. (a) Cones of equal apical angles define equal circles on the spherical surface. (b) Circles project with different radii on the equatorial plane of stereographic projection. (c) Circles project as ellipses with the same area in the equatorial plane of the equal-area projection.

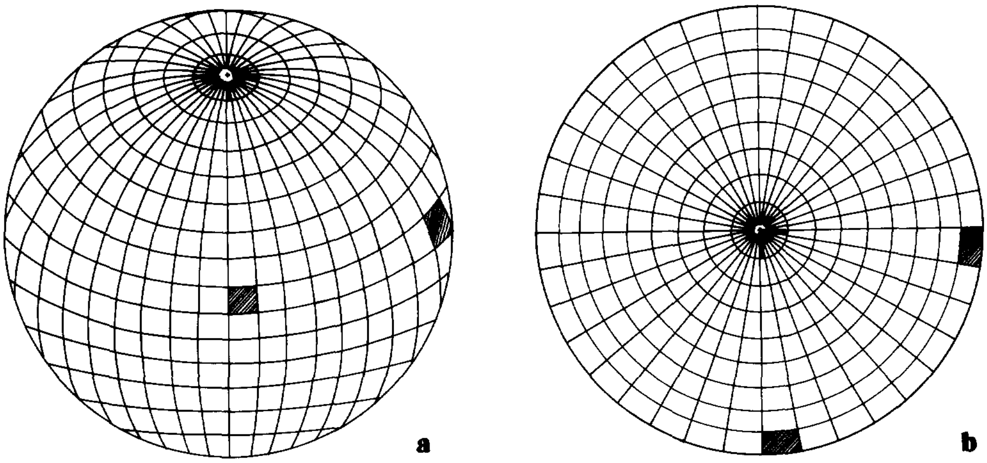

Fig. 2. 2° × 2° areas are not comparable. (a) Triangles and quadrangles have the same measure if expressed in degrees of latitude and longitude, but their actual surface area is different. (b) The Lambert projection makes sure that the surface areas of the triangles and that of quadrangles between the same small circles of latitude remain the same, and assures also that surface areas of triangles and quadrangles are proportional to those on the spherical surface.

CONCLUSIONS

The Schmidt net is constructed using Lambert's azimuthal equal-area projection, in which equal areas on the spherical surface remain equal on the projection plane. The assumption that areas limited by equidistant great and small circles are the same all over the net is a conceptual mistake, quoted and illustrated in some textbooks. The misunderstanding may have arisen from the extrapolation of the E–W property of the equatorial projection (which is a radial property in the polar projection), to the whole equatorial grid, or from the extrapolation of the projection properties to the grid. Triangles and quadrangles of latitudinal–longitudinal sides are neither comparable on the Earth's surface nor on any projection.

The ideal projection, conserving both area and shape does not exist. The Lambert projection (and also the Schmidt net), being equal-area is shape-distorting. This distortion in shape has not always been taken in account in the design of counter holes or counting techniques, so slight errors in the shape and/or value of contour lines can be introduced as a consequence. A second consequence of this shape distortion is that for a given population the shape of the projection changes with rotation of data on the spherical surface. The use of analytical techniques is not affected by errors introduced by the projection itself, so different shapes in the center and the border of the net show the same statistical parameters.

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